
Quantum optics

Marlan O. Scully

Texas A&M University and Max-Planck-Institut für Quantenoptik

M. Suhail Zubairy

Quaid-i-Azam University



CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface	xix
1 Quantum theory of radiation	1
1.1 Quantization of the free electromagnetic field	2
1.1.1 Mode expansion of the field	3
1.1.2 Quantization	4
1.1.3 Commutation relations between electric and magnetic field components	7
1.2 Fock or number states	9
1.3 Lamb shift	13
1.4 Quantum beats	16
1.5 What is light? – The photon concept	20
1.5.1 Vacuum fluctuations and the photon concept	20
1.5.2 Vacuum fluctuations	22
1.5.3 Quantum beats, the quantum eraser, Bell’s theorem, and more	24
1.5.4 ‘Wave function for photons’	24
1.A Equivalence between a many-particle Bose gas and a set of quantized harmonic oscillators	35
<i>Problems</i>	40
<i>References and bibliography</i>	43
2 Coherent and squeezed states of the radiation field	46
2.1 Radiation from a classical current	48
2.2 The coherent state as an eigenstate of the annihilation operator and as a displaced harmonic oscillator state	50
2.3 What is so coherent about coherent states?	51
2.4 Some properties of coherent states	54
2.5 Squeezed state physics	56

2.6	Squeezed states and the uncertainty relation	60
2.7	The squeeze operator and the squeezed coherent states	63
2.7.1	Quadrature variance	65
2.8	Multi-mode squeezing	66
	<i>Problems</i>	67
	<i>References and bibliography</i>	70
3	Quantum distribution theory and partially coherent radiation	72
3.1	Coherent state representation	73
3.1.1	Definition of the coherent state representation	75
3.1.2	Examples of the coherent state representation	77
3.2	Q -representation	79
3.3	The Wigner–Weyl distribution	81
3.4	Generalized representation of the density operator and connection between the P -, Q -, and W -distributions	83
3.5	Q -representation for a squeezed coherent state	86
3.A	Verifying equations (3.1.12a, 3.1.12b)	90
3.B	c -number function correspondence for the Wigner–Weyl distribution	92
	<i>Problems</i>	94
	<i>References and bibliography</i>	96
4	Field–field and photon–photon interferometry	97
4.1	The interferometer as a cosmic probe	98
4.1.1	Michelson interferometer and general relativity	98
4.1.2	The Sagnac ring interferometer	101
4.1.3	Proposed ring laser test of metric gravitation theories	106
4.1.4	The Michelson stellar interferometer	108
4.1.5	Hanbury-Brown–Twiss interferometer	110
4.2	Photon detection and quantum coherence functions	111
4.3	First-order coherence and Young-type double-source experiments	115
4.3.1	Young’s double-slit experiment	115
4.3.2	Young’s experiment with light from two atoms	119
4.4	Second-order coherence	120
4.4.1	The physics behind the Hanbury-Brown–Twiss effect	121
4.4.2	Detection and measurement of squeezed states via homodyne detection	125
4.4.3	Interference of two photons	131
4.4.4	Photon antibunching, Poissonian, and sub-Poissonian light	134
4.5	Photon counting and photon statistics	137

4.A	Classical and quantum descriptions of two-source interference	139
4.B	Calculation of the second-order correlation function	140
	<i>Problems</i>	141
	<i>References and bibliography</i>	143
5	Atom–field interaction – semiclassical theory	145
5.1	Atom–field interaction Hamiltonian	146
5.1.1	Local gauge (phase) invariance and minimal-coupling Hamiltonian	146
5.1.2	Dipole approximation and $\mathbf{r} \cdot \mathbf{E}$ Hamiltonian	148
5.1.3	$\mathbf{p} \cdot \mathbf{A}$ Hamiltonian	149
5.2	Interaction of a single two-level atom with a single-mode field	151
5.2.1	Probability amplitude method	151
5.2.2	Interaction picture	155
5.2.3	Beyond the rotating-wave approximation	158
5.3	Density matrix for a two-level atom	160
5.3.1	Equation of motion for the density matrix	161
5.3.2	Two-level atom	162
5.3.3	Inclusion of elastic collisions between atoms	163
5.4	Maxwell–Schrödinger equations	164
5.4.1	Population matrix and its equation of motion	165
5.4.2	Maxwell’s equations for slowly varying field functions	166
5.5	Semiclassical laser theory	168
5.5.1	Basic principle	169
5.5.2	Lamb’s semiclassical theory	169
5.6	A physical picture of stimulated emission and absorption	173
5.7	Time delay spectroscopy	174
5.A	Equivalence of the $\mathbf{r} \cdot \mathbf{E}$ and the $\mathbf{p} \cdot \mathbf{A}$ interaction Hamiltonians	178
5.A.1	Form-invariant physical quantities	178
5.A.2	Transition probabilities in a two-level atom	180
5.B	Vector model of the density matrix	183
5.C	Quasimode laser physics based on the modes of the universe	185
	<i>Problems</i>	187
	<i>References and bibliography</i>	190
6	Atom–field interaction – quantum theory	193
6.1	Atom–field interaction Hamiltonian	194

6.2	Interaction of a single two-level atom with a single-mode field	196
6.2.1	Probability amplitude method	197
6.2.2	Heisenberg operator method	202
6.2.3	Unitary time-evolution operator method	204
6.3	Weisskopf–Wigner theory of spontaneous emission between two atomic levels	206
6.4	Two-photon cascades	210
6.5	Excitation probabilities for single and double photo-excitation events	213
	<i>Problems</i>	215
	<i>References and bibliography</i>	217
7	Lasing without inversion and other effects of atomic coherence and interference	220
7.1	The Hanle effect	221
7.2	Coherent trapping – dark states	222
7.3	Electromagnetically induced transparency	225
7.4	Lasing without inversion	230
7.4.1	The LWI concept	230
7.4.2	The laser physics approach to LWI: simple treatment	232
7.4.3	LWI analysis	233
7.5	Refractive index enhancement via quantum coherence	236
7.6	Coherent trapping, lasing without inversion, and electromagnetically induced transparency via an exact solution to a simple model	241
	<i>Problems</i>	244
	<i>References and bibliography</i>	245
8	Quantum theory of damping – density operator and wave function approach	248
8.1	General reservoir theory	249
8.2	Atomic decay by thermal and squeezed vacuum reservoirs	250
8.2.1	Thermal reservoir	251
8.2.2	Squeezed vacuum reservoir	253
8.3	Field damping	255
8.4	Fokker–Planck equation	256
8.5	The ‘quantum jump’ approach to damping	260
8.5.1	Conditional density matrices and the null measurement	261
8.5.2	The wave function Monte Carlo approach to damping	263

<i>Problems</i>	267
<i>References and bibliography</i>	269
9 Quantum theory of damping – Heisenberg–Langevin approach	271
9.1 Simple treatment of damping via oscillator reservoir: Markovian white noise	272
9.2 Extended treatment of damping via atom and oscillator reservoirs: non-Markovian colored noise	276
9.2.1 An atomic reservoir approach	276
9.2.2 A generalized treatment of the oscillator reservoir problem	278
9.3 Equations of motion for the field correlation functions	281
9.4 Fluctuation–dissipation theorem and the Einstein relation	283
9.5 Atom in a damped cavity	284
<i>Problems</i>	289
<i>References and bibliography</i>	290
10 Resonance fluorescence	291
10.1 Electric field operator for spontaneous emission from a single atom	292
10.2 An introduction to the resonance fluorescence spectrum	293
10.2.1 Weak driving field limit	293
10.2.2 The strong field limit: sidebands appear	295
10.2.3 The widths of the three peaks in the very strong field limit	296
10.3 Theory of a spectrum analyzer	298
10.4 From single-time to two-time averages: the Onsager– Lax regression theorem	300
10.5 The complete resonance fluorescence spectrum	302
10.5.1 Weak field limit	305
10.5.2 Strong field limit	305
10.6 Photon antibunching	307
10.7 Resonance fluorescence from a driven V system	309
10.A Electric field operator in the far-zone approximation	311
10.B The equations of motion for and exact solution of the density matrix in a dressed-state basis	316
10.B.1 Deriving the equation of motion in the dressed-state basis	316
10.B.2 Solving the equations of motion	317

10.C	The equations of motion for and exact solution of the density matrix in the bare-state basis	320
10.D	Power spectrum in the stationary regime	321
10.E	Derivation of Eq. (10.7.5)	322
	<i>Problems</i>	323
	<i>References and bibliography</i>	325
11	Quantum theory of the laser – density operator approach	327
11.1	Equation of motion for the density matrix	328
11.2	Laser photon statistics	336
11.2.1	Linear approximation ($\mathcal{B} = 0$)	337
11.2.2	Far above threshold ($\mathcal{A} \gg \mathcal{C}$)	338
11.2.3	Exact solution	338
11.3	P -representation of the laser	340
11.4	Natural linewidth	341
11.4.1	Phase diffusion model	342
11.4.2	Fokker–Planck equation and laser linewidth	345
11.5	Off-diagonal elements and laser linewidth	346
11.6	Analogy between the laser threshold and a second-order phase transition	349
11.A	Solution of the equations for the density matrix elements	352
11.B	An exact solution for the P -representation of the laser	354
	<i>Problems</i>	358
	<i>References and bibliography</i>	360
12	Quantum theory of the laser – Heisenberg–Langevin approach	362
12.1	A simple Langevin treatment of the laser linewidth including atomic memory effects	362
12.2	Quantum Langevin equations	367
12.3	c -number Langevin equations	373
12.4	Photon statistics and laser linewidth	376
	<i>Problems</i>	380
	<i>References and bibliography</i>	381
13	Theory of the micromaser	383
13.1	Equation of motion for the field density matrix	384
13.2	Steady-state photon statistics	386
13.3	Preparation of number state in a high- Q micromaser	389
13.3.1	State reduction	390
13.3.2	Trapping states	393
13.4	Linewidth of a micromaser	396

<i>Problems</i>	398
<i>References and bibliography</i>	400
14 Correlated emission laser: concept, theory, and analysis	402
14.1 Correlated spontaneous emission laser concept	403
14.2 Hanle effect correlated emission laser via density matrix analysis	405
14.3 Quantum beat laser via pictorial treatment	413
14.4 Holographic laser	418
14.5 Quantum phase and amplitude fluctuations	423
14.6 Two-photon correlated emission laser	426
14.6.1 Theory	426
14.6.2 Heuristic account of a two-photon CEL	430
14.A Spontaneous emission noise in the quantum beat laser	433
<i>Problems</i>	437
<i>References and bibliography</i>	440
15 Phase sensitivity in quantum optical systems: applications	442
15.1 The CEL gyro	442
15.2 Linear amplification process: general description	446
15.3 Phase-insensitive amplification in a two-level system	448
15.4 Phase-sensitive amplification via the two-photon CEL: noise-free amplification	450
15.5 Laser with an injected squeezed vacuum	452
15.A Analysis of the CEL gyro with reinjection	454
<i>Problems</i>	457
<i>References and bibliography</i>	458
16 Squeezing via nonlinear optical processes	460
16.1 Degenerate parametric amplification	460
16.2 Squeezing in an optical parametric oscillator	463
16.3 Squeezing in the output of a cavity field	467
16.4 Four-wave mixing	471
16.4.1 Amplification and oscillation in four-wave mixing	471
16.4.2 Squeezing in four-wave mixing	475
16.A Effect of pump phase fluctuations on squeezing in degenerate parametric amplification	476
16.B Quantized field treatment of input–output formalism leading to Eq. (16.3.4)	480
<i>Problems</i>	482
<i>References and bibliography</i>	484

17 Atom optics	487
17.1 Mechanical effects of light	488
17.1.1 Atomic deflection	488
17.1.2 Laser cooling	489
17.1.3 Atomic diffraction	490
17.1.4 Semiclassical gradient force	493
17.2 Atomic interferometry	494
17.2.1 Atomic Mach–Zehnder interferometer	494
17.2.2 Atomic gyroscope	496
17.3 Quantum noise in an atomic interferometer	498
17.4 Limits to laser cooling	499
17.4.1 Recoil limit	499
17.4.2 Velocity selective coherent population trapping	501
<i>Problems</i>	503
<i>References and bibliography</i>	504
18 The EPR paradox, hidden variables, and Bell’s theorem	507
18.1 The EPR ‘paradox’	508
18.2 Bell’s inequality	513
18.3 Quantum calculation of the correlations in Bell’s theorem	515
18.4 Hidden variables from a quantum optical perspective	520
18.5 Bell’s theorem without inequalities: Greenberger–Horne–Zeilinger (GHZ) equality	529
18.6 Quantum cryptography	531
18.6.1 Bennett–Brassard protocol	531
18.6.2 Quantum cryptography based on Bell’s theorem	532
18.A Quantum distribution function for a single spin-up particle	533
18.B Quantum distribution function for two particles	534
<i>Problems</i>	536
<i>References and bibliography</i>	539
19 Quantum nondemolition measurements	541
19.1 Conditions for QND measurements	542
19.2 QND measurement of the photon number via the optical Kerr effect	543
19.3 QND measurement of the photon number by dispersive atom–field coupling	547
19.4 QND measurements in optical parametric processes	554
<i>Problems</i>	558
<i>References and bibliography</i>	560

20 Quantum optical tests of complementarity	561
20.1 A micromaser which-path detector	564
20.2 The resonant interaction of atoms with a microwave field and its effect on atomic center-of-mass motion	566
20.3 Quantum eraser	568
20.4 Quantum optical Ramsey fringes	573
20.A Effect of recoil in a micromaser which-path detector	576
<i>Problems</i>	579
<i>References and bibliography</i>	580
21 Two-photon interferometry, the quantum measurement problem, and more	582
21.1 The field–field correlation function of light scattered from two atoms	582
21.1.1 Correlation function $G^{(1)}(\mathbf{r}, t)$ generated by scattering from two excited atoms	585
21.1.2 Excitation by laser light	585
21.1.3 Using three atomic levels as a which-path flag	586
21.2 The field–field and photon–photon correlations of light scattered from two multi-level atoms: quantum eraser	587
21.2.1 Alternative photon basis	590
21.3 Bell’s inequality experiments via two-photon correlations	592
21.4 Two-photon cascade interferometry	595
21.4.1 Two-photon correlations produced by atomic cascade emission	595
21.4.2 Franson–Chiao interferometry	597
21.5 Two-particle interferometry via nonlinear down-conversion and momentum selected photon pairs	600
21.5.1 Two-site down-conversion interferometry	601
21.6 A vacuum–fluctuation picture of the ZWM experiment	607
21.7 High-resolution spectroscopy via two-photon cascade interferometry	610
21.A Scattering from two atoms via an operator approach	614
21.B Calculation of the two-photon correlation function in atomic cascade emission	616
21.C Calculation of the joint count probability in Franson–Chiao interferometry	618

<i>Problems</i>	621
<i>References and bibliography</i>	622
Index	624