

# Laser Electronics

THIRD EDITION

JOSEPH T. VERDEYEN

*Department of Electrical and Computer Engineering  
University of Illinois at Urbana-Champaign, Urbana, Illinois*

PRENTICE HALL SERIES IN SOLID STATE PHYSICAL ELECTRONICS  
Nick Holonyak, Jr., Series Editor



**PRENTICE HALL**  
**Englewood Cliffs, New Jersey 07632**

**Library of Congress Cataloging-in-Publication Data**

Verdeyen, Joseph Thomas

Laser electronics. / Joseph T. Verdeyen. — 3rd ed.

p. cm. — (Prentice Hall series in solid state physical electronics)

Includes bibliographical references and index.

ISBN 0-13-706666-X

1. Lasers. 2. Semiconductor lasers. I. Title. II. Series.

TA1675.V47 1995

621.36'61—dc20 93-2184

CIP

Acquisitions editor: **Alan Apt**

Production editor: **Irwin Zucker**

Copy editor: **Michael Schwartz**

Production coordinator: **Linda Behrens**

Supplements editor: **Alice Dworkin**

Cover design: **Design Solutions**

Cover illustration: **Dr. R. P. Bryan of Photonics Research**

Editorial assistant: **Shirley McGuire**



© 1995, 1989, 1981 by Prentice-Hall, Inc.

A Paramount Communications Company

Englewood Cliffs, New Jersey 07632

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2

**ISBN 0-13-706666-X**

Prentice-Hall International (UK) Limited, *London*

Prentice-Hall of Australia Pty. Limited, *Sydney*

Prentice-Hall Canada Inc., *Toronto*

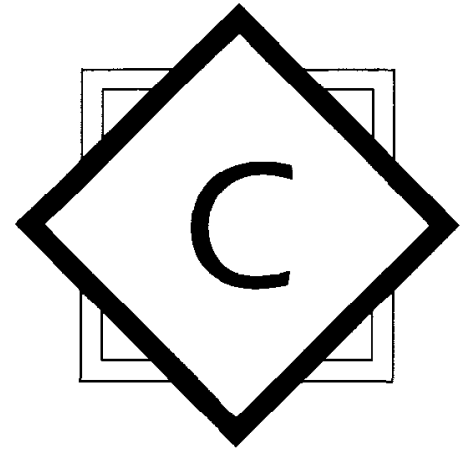
Prentice-Hall Hispanoamericana, S.A., *Mexico*

Prentice-Hall of India Private Limited, *New Delhi*

Prentice-Hall of Japan, Inc., *Tokyo*

Simon & Schuster Asia Pte. Ltd., *Singapore*

Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*



# Contents

<b>List of Symbols</b>	<b>xx</b>
<b>0 Preliminary Comments</b>	<b>1</b>
Note to the students	3
References	6
<b>1 Review of Electromagnetic Theory</b>	<b>8</b>
1.1 Introduction	8
1.2 Maxwell's Equations	9
1.3 Wave Equation for Free Space	10
1.4 Algebraic Form of Maxwell's Equations	11
1.5 Waves in Dielectrics	12
1.6 The Uncertainty Relationships	13
1.7 Spreading of an Electromagnetic Beam	15

1.8	Wave Propagation in Anisotropic Media	16
1.9	Elementary Boundary Value Problems in Optics	20
	1.9.1 Snell's Law,	20
	1.9.2 Brewster's Angle,	21
1.10	Coherent Electromagnetic Radiation	23
1.11	Example of Coherence Effects	28
	Problems	31
	References and Suggested Readings	34

## **2 Ray Tracing in an Optical System**

**35**

2.1	Introduction	35
2.2	Ray Matrix	35
2.3	Some Common Ray Matrices	37
2.4	Applications of Ray Tracing: Optical Cavities	39
2.5	Stability: Stability Diagram	42
2.6	The Unstable Region	44
2.7	Example of Ray Tracing in a Stable Cavity	44
2.8	Repetitive Ray Paths	47
2.9	Initial Conditions: Stable Cavities	48
2.10	Initial Conditions: Unstable Cavities	49
2.11	Astigmatism	50
2.12	Continuous Lens-Like Media	51
	2.12.1 Propagation of a Ray in an Inhomogeneous Medium,	53
	2.12.2 Ray Matrix for a Continuous Lens,	54
2.13	Wave Transformation by a Lens	56
	Problems	57
	References and Suggested Readings	62

## **3 Gaussian Beams**

**63**

3.1	Introduction	63
3.2	Preliminary Ideas: TEM Waves	63
3.3	Lowest-Order TEM <sub>0,0</sub> Mode	66

- 3.4 Physical Description of  $TEM_{0,0}$  Mode 70
  - 3.4.1 Amplitude of the Field, 70
  - 3.4.2 Longitudinal Phase Factor, 71
  - 3.4.3 Radial Phase Factor, 72
- 3.5 Higher-Order Modes 73
- 3.6  $ABCD$  law for Gaussian beams 76
- 3.7 Divergence of the Higher-Order Modes: Spatial Coherence 79
  - Problems 80
  - References and Suggested Readings 84

## 4 Guided Optical Beams

86

- 4.1 Introduction 86
- 4.2 Optical Fibers and Heterostructures: A Slab Waveguide Model 87
  - 4.2.1 Zig-Zag Analysis, 87
  - 4.2.2 Numerical Aperture, 89
- 4.3 Modes in a Step-Index Fiber (or a Heterojunction Laser):  
Wave Equation Approach 90
  - 4.3.1  $TE$  Mode ( $E_z = 0$ ), 92
  - 4.3.2  $TM$  Modes ( $H_z = 0$ ), 94
  - 4.3.3 Graphic Solution for the Propagation Constant: “ $R$ ” and “ $V$ ”  
Parameters, 95
- 4.4 Gaussian Beams in Graded Index (GRIN) Fibers and Lenses 96
- 4.5 Perturbation Theory 102
- 4.6 Dispersion and Loss in Fibers: Data 105
- 4.7 Pulse Propagation in Dispersive Media: Theory 109
- 4.8 Optical Solitons 116
  - Problems 122
  - References and Suggested Readings 127

## 5 Optical Cavities

130

- 5.1 Introduction 130
- 5.2 Gaussian Beams in Simple Stable Resonators 130
- 5.3 Application of the  $ABCD$  Law to Cavities 133
- 5.4 Mode Volume in Stable Resonators 137

Problems	139
References and Suggested Readings	142

## **6 Resonant Optical Cavities** **144**

6.1	General Cavity Concepts	144
6.2	Resonance	144
6.3	Sharpness of Resonance: $Q$ and Finesse	148
6.4	Photon Lifetime	151
6.5	Resonance of the Hermite-Gaussian Modes	154
6.6	Diffraction Losses	156
6.7	Cavity With Gain: An Example	157
	Problems	159
	References and Suggested Readings	170

## **7 Atomic Radiation** **172**

7.1	Introduction and Preliminary Ideas	172
7.2	Blackbody Radiation Theory	173
7.3	Einstein's Approach: $A$ and $B$ Coefficients	179
	7.3.1 Definition of Radiative Processes,	179
	7.3.2 Relationship Between the Coefficients,	181
7.4	Line Shape	183
7.5	Amplification by an Atomic System	187
7.6	Broadening of Spectral Lines	191
	7.6.1 Homogeneous broadening mechanisms,	191
	7.6.2 Inhomogeneous Broadening,	196
	7.6.3 General Comments on the Line Shape,	200
7.7	Review	200
	Problems	201
	References and Suggested Readings	205

## **8 Laser Oscillation and Amplification** **207**

8.1	Introduction: Threshold Condition for Oscillation	207
-----	---	-----

8.2	Laser Oscillation and Amplification in a Homogeneous Broadened Transition	208
8.3	Gain Saturation in a Homogeneous Broadened Transition	212
8.4	Laser Oscillation in an Inhomogeneous System	223
8.5	Multimode Oscillation	229
8.6	Gain Saturation in Doppler-Broadened Transition: Mathematical Treatment	230
8.7	Amplified Spontaneous Emission (ASE)	234
8.8	Laser Oscillation: A Different Viewpoint	238
	Problems	242
	References and Suggested Readings	258

## 9 General Characteristics of Lasers

260

9.0	Introduction	260
9.1	Limiting Efficiency	260
	9.1.1 Factors in the efficiency,	260
	9.1.2 Two, 3, 4 : : , $n$ level lasers,	261
9.2	CW Laser	263
	9.2.1 Traveling Wave Ring Laser,	264
	9.2.2 Optimum Coupling,	267
	9.2.3 Standing Wave Lasers,	269
9.3	Laser Dynamics	274
	9.3.1 Introduction and model,	274
	9.3.2 Case a: A sub-threshold system,	276
	9.3.3 Case b: A CW laser: threshold conditions,	276
	9.3.4 Case c: A sinusoidal modulated pump,	277
	9.3.5 Case d. A sudden "step" change in excitation rate,	280
	9.3.6 Case e: Pulsed excitation $\rightarrow$ gain switching,	282
9.4	$Q$ Switching, $Q$ Spoiling, or Giant Pulse Lasers	284
9.5	Mode Locking	296
	9.5.1 Preliminary considerations,	296
	9.5.2 Mode locking in an inhomogeneous broadened laser,	298
	9.5.3 Active mode locking,	304
9.6	Pulse Propagation in Saturable Amplifiers or Absorbers	311
9.7	Saturable Absorber (Colliding Pulse) Mode Locking	317

- 9.8 Additive-Pulse Mode Locking 322
- Problems 324
- References and Suggested Readings 344

## **10 Laser Excitation 347**

- 10.1 Introduction 347
- 10.2 Three- and Four-Level Lasers 348
- 10.3 Ruby Lasers 351
- 10.4 Rare Earth Lasers and Amplifiers 358
  - 10.4.1 *General Considerations*, 358
  - 10.4.2 *Nd:YAG lasers: Data*, 359
  - 10.4.3 *Nd:YAG Pumped by a Semiconductor Laser*, 362
  - 10.4.4 *Neodymium-Glass Lasers*, 369
  - 10.4.6 *Erbium-Doped-Fiber-Amplifiers*, 371
- 10.5 Broad-Band Optical Gain 376
  - 10.5.1 *Band-to-Band Emission and Absorption*, 376
  - 10.5.2 *Theory of Band-to-Band Emission and Absorption*, 377
- 10.6 Tunable Lasers 385
  - 10.6.1 *General Considerations*, 385
  - 10.6.2 *Dye Lasers*, 386
  - 10.6.3 *Tunable Solid State Lasers*, 391
  - 10.6.4 *Cavities for Tunable Lasers*, 395
- 10.7 Gaseous-Discharge Lasers 396
  - 10.7.1 *Overview*, 396
  - 10.7.2 *Helium-Neon Laser*, 397
  - 10.7.3 *Ion Lasers*, 403
  - 10.7.4 *CO<sub>2</sub> Lasers*, 405
- 10.8 Excimer Lasers: General Considerations 411
  - 10.8.1 *Formation of the Excimer State*, 412
  - 10.8.2 *Excitation of the Rare Gas-Halogen Excimer Lasers*, 415
- 10.9 Free Electron Laser 417
  - Problems 423
  - References and Suggested Readings 434

## **11 Semiconductor Lasers 440**

- 11.1 Introduction 440

	11.1.1	Overview, 440	
	11.1.2	Populations in Semiconductor Laser, 442	
11.2		Review of Elementary Semiconductor Theory	444
	11.2.1	Density of States, 445	
11.3		Occupation Probability: Quasi-Fermi Levels	449
11.4		Optical Absorption and Gain in a Semiconductor	450
	11.4.1	Gain Coefficient in a Semiconductor, 454	
	11.4.2	Spontaneous Emission Profile, 459	
	11.4.3	An Example of an Inverted Semiconductor, 460	
11.5		Diode Laser	464
	11.5.1	Homojunction Laser, 464	
	11.5.2	Heterojunction Lasers, 467	
11.6		Quantum Size Effects	470
	11.6.1	Infinite Barriers, 470	
	11.6.2	Finite Barriers: An Example, 476	
11.7		Vertical Cavity Surface Emitting Lasers	482
11.8		Modulation of Semiconductor Lasers	486
	11.8.1	Static Characteristics, 488	
	11.8.2	Frequency Response of Diode Lasers, 489	
		Problems	492
		References and Suggested Readings	499

## **12 Advanced Topics in Laser Electromagnetics**

**502**

12.1		Introduction	502
12.2		Semiconductor Cavities	503
	12.2.1	TE Modes ( $E_z = 0$ ), 505	
	12.2.2	TM Modes ( $H_z = 0$ ), 507	
	12.2.3	Polarization of TE and TM Modes, 508	
12.3		Gain Guiding: An Example	509
12.4		Optical Confinement and Effective Index	516
12.5		Distributed Feedback and Bragg Reflectors	517
	12.5.1	Introduction, 517	
	12.5.2	Coupled Mode Analysis, 520	
	12.5.3	Distributed Bragg Reflector, 524	
	12.5.4	A Quarter-Wave Bandpass Filter, 525	

	12.5.5	<i>Distributed Feedback Lasers (Active Mirrors)</i> , 528
	12.5.6	<i>Tunable Semiconductor Lasers</i> , 531
12.6		Unstable Resonators 534
	12.6.1	<i>General Considerations</i> , 534
	12.6.2	<i>Unstable Confocal Resonator</i> , 540
12.7		Integral Equation Approach to Cavities 543
	12.7.1	<i>Mathematical Formulation</i> , 543
	12.7.2	<i>Fox and Li Results</i> , 547
	12.7.3	<i>Stable Confocal Resonator</i> , 550
12.8		Field Analysis of Unstable Cavities 555
12.9		<i>ABCD</i> Law for “Tapered Mirror” Cavities 562
12.10		Laser Arrays 568
	12.10.1	<i>System Considerations</i> , 568
	12.10.2	<i>Semiconductor Laser Array: Physical Picture</i> , 568
	12.10.3	<i>Supermodes of the Array</i> , 570
	12.10.4	<i>Radiation Pattern</i> , 574
		Problems 574
		References and Suggested Readings 585

### **13 Maxwell’s Equations and the “Classical” Atom**

**589**

13.1		Introduction 589
13.2		Polarization Current 590
13.3		Wave Propagation With Active Atoms 592
13.4		The Classical $A_{21}$ Coefficient 596
13.5		(Slater) Modes of a Laser 597
	13.5.1	<i>Slater Modes of a Lossless Cavity</i> , 598
	13.5.2	<i>Lossy Cavity With a Source</i> , 600
13.6		Dynamics of the Fields 602
	13.6.1	<i>Excitation Clamped to Zero</i> , 602
	13.6.2	<i>Time Evolution of the Field</i> , 603
13.7		Summary 609
		Problems 610
		References and Suggested Readings 615

**14 Quantum Theory of the Field-Atom Interaction 616**

- 14.1 Introduction 616
- 14.2 Schrödinger Description 617
- 14.3 Derivation of the Einstein Coefficients 621
- 14.4 Dynamics of an Isolated Atom 624
- 14.5 Density Matrix Approach 627
  - 14.5.1 Introduction, 627
  - 14.5.2 Definition, 628
- 14.6 Equation of Motion for the Density Matrix 633
- 14.7 Two-Level System 635
- 14.8 Steady State Polarization Current 639
- 14.9 Multilevel or Multiphoton Phenomena 643
- 14.10 Raman Effects 651
  - 14.10.1 Phenomena, 651
  - 14.10.2 A Classical Analysis of the Raman Effect., 654
  - 14.10.3 Density Matrix Description of the Raman Effect, 660
- 14.11 Propagation of Pulses: Self-Induced Transparency 665
  - 14.11.1 Motivation for the Analysis, 665
  - 14.11.2 A Self-Consistent Analysis of the Field-Atom Interaction, 666
  - 14.11.3 "Area" Theorem, 670
  - 14.11.4 Pulse Solution, 673
- Problems 676
- References and Suggested Readings 679

**15 Spectroscopy of Common Lasers 681**

- 15.1 Introduction 681
- 15.2 Atomic Notation 681
  - 15.2.1 Energy Levels, 681
  - 15.2.2 Transitions: Selection Rules, 682
- 15.3 Molecular Structure: Diatomic Molecules 684
  - 15.3.1 Preliminary Comments, 684
  - 15.3.2 Rotational Structure and Transitions, 685
  - 15.3.3 Thermal Distribution of the Population in Rotational States, 686
  - 15.3.4 Vibrational Structure, 687

- 15.3.5 *Vibration-Rotational Transitions*, 688
- 15.3.6 *Relative Gain on P and R Branches: Partial and Total Inversions*, 689

## 15.4 Electronic States in Molecules 691

- 15.4.1 *Notation*, 691
- 15.4.2 *The Franck–Condon Principle*, 692
- 15.4.3 *Molecular Nitrogen Lasers\**, 692

Problems 693

References and Suggested Readings 695

# 16 Detection of Optical Radiation 697

## 16.1 Introduction 697

## 16.2 Quantum Detectors 697

- 16.2.1 *Vacuum Photodiode*, 698
- 16.2.2 *Photomultiplier*, 699

## 16.3 Solid-State Quantum Detectors 701

- 16.3.1 *Photoconductor*, 701
- 16.3.2 *Junction Photodiode*, 703
- 16.3.3 *p-i-n Diode*, 706
- 16.3.4 *Avalanche Photodiode*, 707

## 16.4 Noise Considerations 707

## 16.5 Mathematics of Noise 709

## 16.6 Sources of Noise 713

- 16.6.1 *Shot Noise*, 713
- 16.6.2 *Thermal Noise*, 714
- 16.6.3 *Noise Figure of Video Amplifiers*, 716
- 16.6.4 *Background Radiation*, 717

## 16.7 Limits of Detection Systems 718

- 16.7.1 *Video Detection of Photons*, 718
- 16.7.2 *Heterodyne System*, 722

Problems 725

References and Suggested Readings 728

# 17 Gas-Discharge Phenomena 729

## 17.1 Introduction 729

## 17.2 Terminal Characteristics 731

17.3	Spatial Characteristics	732
17.4	Electron Gas	734
	17.4.1 Background,	734
	17.4.2 “Average” or “Typical” Electron,	734
	17.4.3 Electron Distribution Function,	741
	17.4.4 Computation of Rates,	743
	17.4.5 Computation of a Flux,	745
17.5	Ionization Balance	746
17.6	Example of Gas-Discharge Excitation of a CO <sub>2</sub> Laser	748
	17.6.1 Preliminary Information,	748
	17.6.2 Experimental Detail and Results,	748
	17.6.3 Theoretical Calculations,	750
	17.6.4 Correlation Between Experiment and Theory,	753
	17.6.5 Laser-Level Excitation,	756
17.7	Electron Beam Sustained Operation	758
	Problems	761
	References and Suggested Readings	764

## Appendices

<b>I</b>	<b>An Introduction to Scattering Matrices</b>	<b>765</b>
<b>II</b>	<b>Detailed Balancing or Microscopic Reversibility</b>	<b>770</b>
<b>III</b>	<b>The Kramers–Kronig Relations</b>	<b>774</b>
	<b>Index</b>	<b>779</b>